$$\begin{array}{cccc} x & y & z & = c \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \begin{array}{l} x+3z=5 \\ y-2z=1 \end{array} \longrightarrow \begin{array}{l} x=-3z+5 \\ y=2z+1 \\ z=z \end{array}$$

RREF

$$\left.\begin{array}{l} x=-3t+5 \\ y=2t+1 \\ z=t \end{array}\right\} \text{PARAMETRIC SOLUTION}$$

$t \in (-\infty, \infty)$

INFINITELY MANY SOLUTIONS

8.2
~~8.1~~ MATRICES

RECTANGULAR ARRAY OF NUMBERS

$$\begin{array}{l} \text{ROW 1} \rightarrow \\ \text{ROW 2} \rightarrow \\ \text{ROW 3} \rightarrow \end{array} \begin{bmatrix} 3 & 4 & -11 & -17 \\ 2 & 1 & -4 & 5 \\ -1 & -2 & 5 & -9 \end{bmatrix} = A$$

COLUMN 1, COLUMN 2, COLUMN 3, COLUMN 4

A IS A 3×4 MATRIX

a_{ij} = ENTRY IN ROW i, COLUMN j

$a_{2,3}$ = ENTRY IN ROW 2, COLUMN 3 $= -4$

$a_{1,4} = -17$

ORDER OF MATRIX A

IS #ROWS IN A $\times$ #COLUMNS IN A

"BY"

SCALAR MULTIPLICATION ($k \in \mathbb{R}$)

$$kA = [ka_{ij}]$$

$$A = \begin{bmatrix} 1 & 3 \\ 7 & 8 \\ 6 & 2 \end{bmatrix} \qquad 3A = \begin{bmatrix} 3\cdot 1 & 3\cdot 3 \\ 3\cdot 7 & 3\cdot 8 \\ 3\cdot 6 & 3\cdot 2 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 21 & 24 \\ 18 & 6 \end{bmatrix}$$

PROPERTIES

$$A + B = B + A$$

$$(A+B)+C = A+(B+C)$$

$$k_1(k_2A) = (k_1k_2)A$$

$$1A = A$$

$$c(A+B) = cA + cB$$

$$(c+d)A = cA + dA$$

$\cancel{A2}$

$2A$

$$\underset{m\times n}{O} + \underset{m\times n}{A} = A$$

$$O_{m\times n}$$

$$O = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{n \text{ COLUMNS}} \left.\vphantom{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}\right\} m \text{ ROWS}$$

$A = B$ IF $\begin{cases} A, B \text{ HAVE THE SAME ORDER AND} \\ a_{ij} = b_{ij} \text{ FOR ALL } i, j \end{cases}$

$\underset{2\times2}{A} = \begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$ $\underset{2\times2}{B} = \begin{bmatrix} \sqrt[3]{8} & 1-4 \\ 0 & 2^2 \end{bmatrix}$ $\underset{2\times3}{C} = \begin{bmatrix} 2 & -3 & 0 \\ 0 & 4 & 0 \end{bmatrix}$

$a_{11} = 2 \quad b_{11} = \sqrt[3]{8} = 2$

$a_{12} = -3 \quad b_{12} = 1-4 = -3$

$a_{21} = 0 \quad b_{21} = 0$

$a_{22} = 4 \quad b_{22} = 2^2 = 4$

$A = B$

$a_{11} = c_{11}$

$a_{12} = c_{12}$

$a_{21} = c_{21}$

$a_{22} = c_{22}$

$A \neq C$

MATRIX ADDITION

IF A, B HAVE THE SAME ORDER

$A + B = [a_{ij} + b_{ij}]$ ← THE MATRIX WHOSE ENTRY IN ROW i, COLUMN j IS $a_{ij} + b_{ij}$

$A = \begin{bmatrix} 1 & 3 & 2 \\ 5 & 4 & 6 \end{bmatrix}$

$B = \begin{bmatrix} 0 & -2 & -5 \\ 1 & -4 & 7 \end{bmatrix}$

$A + B = \begin{bmatrix} 1+0 & 3+-2 & 2+-5 \\ 5+1 & 4+-4 & 6+7 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -3 \\ 6 & 0 & 13 \end{bmatrix} = B + A$

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

IF $3A + 2X = B$, FIND X

$$2X = -3A + B$$

$$X = \tfrac{1}{2}(-3A + B)$$

$$= \tfrac{1}{2}\left(-3\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}\right)$$

$$= \tfrac{1}{2}\left(\begin{bmatrix} -9 & -6 \\ -12 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}\right)$$

$$= \tfrac{1}{2}\begin{bmatrix} -8 & -2 \\ -9 & -1 \end{bmatrix}$$

$$X = \begin{bmatrix} -4 & -1 \\ -\tfrac{9}{2} & -\tfrac{1}{2} \end{bmatrix}$$

CHECK:

$$2X = \begin{bmatrix} -8 & -2 \\ -9 & -1 \end{bmatrix} \qquad 3A + 2X = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} = B$$

$$3A = \begin{bmatrix} 9 & 6 \\ 12 & 3 \end{bmatrix}$$

MATRIX MULTIPLICATION

IF A IS A MATRIX OF ORDER $m \times n$

B IS A MATRIX OF ORDER $n \times p$

THEN AB IS A MATRIX OF ORDER $m \times p$

WHOSE ENTRY IN ROW i, COLUMN j

IS THE "DOT PRODUCT" OF ROW i OF A

WITH COLUMN j OF B

eg.

$$\underset{2\times 3}{A} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & -2 \end{bmatrix} \qquad \underset{3\times 2}{B} = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ -2 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\underset{2\times 3\ \ 3\times 2}{AB} = \begin{bmatrix} \underbrace{2(-1)+1(1)+(-1)(-2)}_{\text{ROW 1, COLUMN 1}} & \underbrace{2(2)+1(0)+(-1)3}_{\text{ROW 1, COLUMN 2}} \\ 0(-1)+(-1)1+(-2)3 & 0(2)+(-1)0+(-2)3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -7 & -6 \end{bmatrix}$$

IS A 2×2 MATRIX

$AB \neq BA$

$$\underset{3\times 2\ \ 2\times 3}{BA} = \begin{bmatrix} -2+0 & -1-2 & 1-4 \\ 2+0 & 1+0 & -1+0 \\ -4+0 & -2-3 & 2-6 \end{bmatrix} = \begin{bmatrix} -2 & -3 & -3 \\ 2 & 1 & -1 \\ -4 & -5 & -4 \end{bmatrix}$$

IS A 3×3 MATRIX

$$(AB)C = A(BC)$$

$$\underbrace{\underbrace{\underset{m\times n}{A}\;\underset{n\times p}{B}}_{m\times p}\;\underset{p\times q}{C}}_{m\times q} \qquad \underbrace{\underset{m\times n}{A}\;\underbrace{\underset{n\times p}{B}\;\underset{p\times q}{C}}_{n\times q}}_{m\times q}$$

$$A(B+C) = AB + AC$$

$$(B+C)A = BA + CA$$